

CHAPTER

7

Supplemental Problems

Gravitation

1. Titan, the largest moon of Saturn, has a mean orbital radius of 1.22×10^9 m. The orbital period of Titan is 15.95 days. Hyperion, another moon of Saturn, orbits at a mean radius of 1.48×10^9 m. Use Kepler's third law of planetary motion to predict the orbital period of Hyperion in days.
2. The mass of Earth is 5.97×10^{24} kg, the mass of the Moon is 7.35×10^{22} kg, and the mean distance of the Moon from the center of Earth is 3.84×10^5 km. Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon.
3. Two identical bowling balls are placed 1.00 m apart. The gravitational force between the bowling balls is 3.084×10^{-9} N.
 - a. Find the mass of a bowling ball.
 - b. Compare the weight of the first ball with the gravitational force exerted on it by the second ball.
4. The planet Mercury travels around the Sun with a mean orbital radius of 5.8×10^{10} m. The mass of the Sun is 1.99×10^{30} kg. Use Newton's version of Kepler's third law to determine how long it takes Mercury to orbit the Sun. Give your answer in Earth days.
5. Io, the closest moon to Jupiter, has a period of 1.77 days and a mean orbital radius of 4.22×10^8 m. Use this information together with Newton's version of Kepler's third law to determine the mass of Jupiter.
6. A satellite is placed in a circular orbit 100.0 km above Earth's surface. Earth's mass is 5.97×10^{24} kg and its average radius is 6.38×10^6 m.
 - a. What is the speed of the satellite?
 - b. How many minutes does it take the satellite to complete one orbit?
7. You have been hired to do calculations for a consortium that plans to place a space station in orbit around Mars. The mass of Mars is 6.42×10^{23} kg and its radius is 3.40×10^6 m. In order for the space station to appear to remain over the same spot on Mars at all times, its orbital period must be equal to the length of a day on Mars: 8.86×10^4 s. At what height above the surface of Mars should the space station be located in order to maintain this orbit? Use Newton's version of Kepler's third law.
8. The asteroid Vesta has a mass of 3.0×10^{20} kg and an average radius of 510 km.
 - a. What is the acceleration due to gravity at its surface?
 - b. How much would a 95-kg astronaut weigh at the surface of Vesta?
9. The Moon has an average radius of 1.74×10^3 km. At the Moon's surface, g_{Moon} has a value of 1.62 m/s². What is the value of the acceleration due to gravity at an altitude of 1.00×10^2 km above the Moon's surface?
10. Use data from Table 7-1 in your textbook.
 - a. Find the Sun's gravitational field strength at Earth's orbit.
 - b. How does this compare with the Sun's gravitational field strength at the orbit of Pluto?
11. Earth has an orbital period of 365 days and its mean distance from the Sun is 1.495×10^8 km. The planet Pluto's mean distance from the Sun is 5.896×10^9 km. Using Kepler's third law, calculate Pluto's orbital period in Earth days.

7 Supplemental Problems*continued*

- 12.** The mass of Earth is 5.98×10^{24} kg and the mass of the Sun is 330,000 times greater than the mass of Earth. If the center of Earth is, on average, 1.495×10^{11} m from the center of the Sun, calculate the magnitude of the gravitational force the Sun exerts on Earth.
- 13.** Two metal spheres, each weighing 24.0 kg are placed 0.0500 m apart. Calculate the magnitude of the gravitational force the two spheres exert on each other.
- 14.** A car and a truck are traveling side by side on the highway. The car has a mass of 1.37×10^3 kg and the truck has a mass of 9.92×10^3 kg. If the cars are separated by 2.10 m, find the force of gravitational attraction between the car and the truck.
- 15.** A satellite is in orbit 3.11×10^6 m from the center of Earth. The mass of Earth is 5.98×10^{24} kg. Calculate the orbital period of the satellite.
- 16.** The planet Venus orbits the Sun with a mean orbital radius of 1.076×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.
- 17.** A satellite is in orbit at a distance of 6750 km from the center of Earth. The mass of Earth is 5.98×10^{24} kg. What is the orbital speed of the satellite?
- 18.** Given that the mass of Earth is 5.98×10^{24} kg, what is the orbital radius of a satellite that has an orbital period of exactly one day (assume that a day is exactly 24 hours in length)?
- 19.** The Moon has a mass of 7.349×10^{22} kg and a radius of 1737 km. How much would a 75.0-kg person weigh standing on the surface of the Moon?
- 20.** Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.145×10^4 km. Earth has a mass of 5.98×10^{24} kg and a radius of 6.38×10^6 m. How many times their Earth weight would a 75.0-kg person weigh when standing on the surface of Jupiter? (Assume Jupiter has a solid surface to stand on.)
- 21.** A 5.0-kg mass weighs 8.1 N on the surface of the Moon. If the radius of the Moon is 1737 km, what is the mass of the Moon?
- 22.** Acceleration due to gravity on Earth's surface is 9.80 m/s^2 . Thus, a 1.00-kg mass weighs 9.80 N on the surface of Earth. If the radius of Earth was cut exactly in half but the mass of Earth remained unchanged, how much would a 1.00-kg mass weigh on the surface of Earth?

Answer Key

Chapter 7

1. Titan, the largest moon of Saturn, has a mean orbital radius of 1.22×10^9 m. The orbital period of Titan is 15.95 days. Hyperion, another moon of Saturn, orbits at a mean radius of 1.48×10^9 m. Use Kepler's third law of planetary motion to predict the orbital period of Hyperion in days.

$$\left(\frac{T_{\text{Hyperion}}}{T_{\text{Titan}}}\right)^2 = \left(\frac{r_{\text{Hyperion}}}{r_{\text{Titan}}}\right)^3$$

$$\begin{aligned} T_{\text{Hyperion}} &= T_{\text{Titan}} \sqrt{\left(\frac{r_{\text{Hyperion}}}{r_{\text{Titan}}}\right)^3} \\ &= (15.95 \text{ days}) \sqrt{\left(\frac{1.48 \times 10^9 \text{ m}}{1.22 \times 10^9 \text{ m}}\right)^3} \\ &= 21.3 \text{ days} \end{aligned}$$

2. The mass of Earth is 5.97×10^{24} kg, the mass of the Moon is 7.35×10^{22} kg, and the mean distance of the Moon from the center of Earth is 3.84×10^5 km. Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon.

$$F = G \frac{m_E m_M}{r^2}$$

$$r = 3.84 \times 10^5 \text{ km} = 3.84 \times 10^8 \text{ m}$$

$$\begin{aligned} F &= 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \frac{(5.97 \times 10^{24} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} \\ &= 1.98 \times 10^{20} \text{ N} \end{aligned}$$

3. Two identical bowling balls are placed 1.00 m apart. The gravitational force between the bowling balls is 3.084×10^{-9} N.
- a. Find the mass of a bowling ball.

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$m_1 = m_2 = m$$

$$F_g = \frac{Gm^2}{r^2}$$

$$\begin{aligned} m &= \sqrt{\frac{F_g r^2}{G}} \\ &= \sqrt{\frac{(3.084 \times 10^{-9} \text{ N})(1.00 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)}} \\ &= 6.80 \text{ kg} \end{aligned}$$

- b. Compare the weight of the first ball with the gravitational force exerted on it by the second ball.

$$F_g = mg = (6.80 \text{ kg})(9.80 \text{ m/s}^2)$$

$$F_{g1} = 66.6 \text{ N}$$

Answer Key

Chapter 7 continued

$$F_{g2} = 3.084 \times 10^{-9} \text{ N}$$

$$\begin{aligned} \frac{F_{g1}}{F_{g2}} &= \frac{66.6 \text{ N}}{3.084 \times 10^{-9} \text{ N}} \\ &= 2.16 \times 10^{10} \end{aligned}$$

4. The planet Mercury travels around the Sun with a mean orbital radius of $5.8 \times 10^{10} \text{ m}$. The mass of the Sun is $1.99 \times 10^{30} \text{ kg}$. Use Newton's version of Kepler's third law to determine how long it takes Mercury to orbit the Sun. Give your answer in Earth days.

$$T^2 = \left(\frac{4\pi^2}{Gm_S} \right) r^3$$

$$T = 2\pi \sqrt{\frac{r^3}{Gm_S}}$$

$$\begin{aligned} &= 2\pi \sqrt{\frac{(5.8 \times 10^{10} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1 \text{ day}}{24 \text{ h}} \right) \\ &= 88 \text{ days} \end{aligned}$$

5. Io, the closest moon to Jupiter, has a period of 1.77 days and a mean orbital radius of $4.22 \times 10^8 \text{ m}$. Use this information together with Newton's version of Kepler's third law to determine the mass of Jupiter.

$$T_{\text{Io}}^2 = \left(\frac{4\pi^2}{Gm_{\text{Jupiter}}} \right) r_{\text{Io}}^3$$

$$m_{\text{Jupiter}} = \left(\frac{4\pi^2}{GT_{\text{Io}}^2} \right) r_{\text{Io}}^3$$

$$\begin{aligned} T_{\text{Io}} &= (1.77 \text{ days}) \left(\frac{24 \text{ h}}{1 \text{ day}} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) \\ &= 1.53 \times 10^5 \text{ s} \end{aligned}$$

$$\begin{aligned} m_{\text{Jupiter}} &= \left(\frac{4\pi^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.53 \times 10^5 \text{ s})^2} \right) (4.22 \times 10^8 \text{ m})^3 \\ &= 1.90 \times 10^{27} \text{ kg} \end{aligned}$$

6. A satellite is placed in a circular orbit 100.0 km above Earth's surface. Earth's mass is $5.97 \times 10^{24} \text{ kg}$ and its average radius is $6.38 \times 10^6 \text{ m}$.
- a. What is the speed of the satellite?

$$h = 100.0 \text{ km} = 1.000 \times 10^5 \text{ m}$$

$$r = r_E + h = 6.38 \times 10^6 \text{ m} + 1.000 \times 10^5 \text{ m} = 6.48 \times 10^6 \text{ m}$$

$$v = \sqrt{\frac{Gm_E}{r}}$$

Answer Key

Chapter 7 continued

$$\begin{aligned} &= \sqrt{\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{(6.48 \times 10^6 \text{ m})}} \\ &= 7.84 \times 10^3 \text{ m/s} \end{aligned}$$

- b. How many minutes does it take the satellite to complete one orbit?

$$\begin{aligned} T &= 2\pi \sqrt{\frac{r^3}{Gm_E}} \\ &= 2\pi \sqrt{\frac{(6.48 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}} \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \\ &= 86.6 \text{ min} \end{aligned}$$

7. You have been hired to do calculations for a consortium that plans to place a space station in orbit around Mars. The mass of Mars is $6.42 \times 10^{23} \text{ kg}$ and its radius is $3.40 \times 10^6 \text{ m}$. In order for the space station to appear to remain over the same spot on Mars at all times, its orbital period must be equal to the length of a day on Mars: $8.86 \times 10^4 \text{ s}$. At what height above the surface of Mars should the space station be located in order to maintain this orbit? Use Newton's version of Kepler's third law.

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{Gm_{\text{Mars}}}\right)r^3 \\ r &= \sqrt[3]{\frac{Gm_{\text{Mars}}T^2}{4\pi^2}} \\ &= \sqrt[3]{\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.42 \times 10^{23} \text{ kg})(8.86 \times 10^4 \text{ s})^2}{4\pi^2}} \\ &= 2.04 \times 10^7 \text{ m} \end{aligned}$$

Height above the surface of Mars:

$$h = r - r_{\text{Mars}} = 2.04 \times 10^7 \text{ m} - 3.40 \times 10^6 \text{ m} = 1.70 \times 10^7 \text{ m}$$

8. The asteroid Vesta has a mass of $3.0 \times 10^{20} \text{ kg}$ and an average radius of 510 km.
- a. What is the acceleration of gravity at its surface?

$$\begin{aligned} g_{\text{Vesta}} &= \frac{Gm_{\text{Vesta}}}{r_{\text{Vesta}}^2} \\ r_{\text{Vesta}} &= 510 \text{ km} = 5.10 \times 10^5 \text{ m} \\ g_{\text{Vesta}} &= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(3.0 \times 10^{20} \text{ kg})}{(5.10 \times 10^5 \text{ m})^2} \\ &= 7.7 \times 10^{-2} \text{ m/s}^2 \end{aligned}$$

Answer Key

Chapter 7 continued

- b. How much would a 95-kg astronaut weigh at the surface of Vesta?

$$\begin{aligned}F_g &= mg_{\text{Vesta}} \\&= (95 \text{ kg})(7.7 \times 10^{-2} \text{ m/s}^2) \\&= 7.3 \text{ N}\end{aligned}$$

9. The Moon has an average radius of 1.74×10^3 km. At the Moon's surface, g_{Moon} has a value of 1.62 m/s^2 . What is the value of the acceleration due to gravity at an altitude of 1.00×10^2 km above the Moon's surface?

$$r = r_{\text{Moon}} + h = 1.74 \times 10^6 \text{ m} + 1.00 \times 10^5 \text{ m} = 1.84 \times 10^6 \text{ m}$$

$$\begin{aligned}a &= g_{\text{Moon}} \left(\frac{r_{\text{Moon}}}{r} \right)^2 \\&= (1.62 \text{ m/s}^2) \left(\frac{1.74 \times 10^6 \text{ m}}{1.84 \times 10^6 \text{ m}} \right)^2 \\&= 1.45 \text{ m/s}^2\end{aligned}$$

10. Use data from Table 7-1 in the text.

- a. Find the Sun's gravitational field strength at Earth's orbit.

$$\begin{aligned}g_{\text{at Earth}} &= \frac{Gm_s}{(r_{\text{Sun to Earth}})^2} \\&= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} \\&= 5.90 \times 10^{-3} \text{ m/s}^2\end{aligned}$$

- b. How does this compare with the Sun's gravitational field strength at the orbit of Pluto?

$$\begin{aligned}g_{\text{at Pluto}} &= \frac{Gm_s}{(r_{\text{Sun to Pluto}})^2} \\&= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(5.87 \times 10^{12} \text{ m})^2} \\&= 3.85 \times 10^{-6} \text{ m/s}^2\end{aligned}$$

$$\begin{aligned}\frac{g_{\text{at Earth}}}{g_{\text{at Pluto}}} &= \frac{5.90 \times 10^{-3} \text{ m/s}^2}{3.85 \times 10^{-6} \text{ m/s}^2} \\&= 1.53 \times 10^3\end{aligned}$$

11. Earth has an orbital period of 365 days and its mean distance from the Sun is 1.495×10^8 km. The planet Pluto's mean distance from the Sun is 5.896×10^9 km. Using Kepler's third law, calculate Pluto's orbital period in Earth days.

$$\left(\frac{T_P}{T_E} \right)^2 = \left(\frac{r_P}{r_E} \right)^3$$

Answer Key

Chapter 7 continued

$$\begin{aligned}T_P &= T_E \sqrt{\left(\frac{r_P}{r_E}\right)^3} \\&= (365 \text{ days}) \sqrt{\left(\frac{5.896 \times 10^9 \text{ km}}{1.495 \times 10^8 \text{ km}}\right)^3} \\&= 9.04 \times 10^4 \text{ days}\end{aligned}$$

12. The mass of Earth is 5.98×10^{24} kg and the mass of the Sun is 330,000 times greater than the mass of Earth. If the center of Earth is, on average, 1.495×10^{11} m from the center of the Sun, calculate the magnitude of the gravitational force the Sun exerts on Earth.

$$\begin{aligned}F &= G \frac{m_1 m_2}{r^2} \\&= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(330,000 \times 5.98 \times 10^{24} \text{ kg})}{(1.495 \times 10^{11} \text{ m})^2} \\&= 3.5 \times 10^{22} \text{ N}\end{aligned}$$

13. Two metal spheres, each weighing 24.0 kg are placed 0.0500 m apart. Calculate the magnitude of the gravitational force the two spheres exert on each other.

$$\begin{aligned}F &= G \frac{m_1 m_2}{r^2} \\&= (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) \frac{(24.0 \text{ kg})(24.0 \text{ kg})}{(0.0500 \text{ m})^2} \\&= 1.54 \times 10^{-5} \text{ N}\end{aligned}$$

14. A car and a truck are traveling side by side on the highway. The car has a mass of 1.37×10^3 kg and the truck has a mass of 9.92×10^3 kg. If the cars are separated by 2.10 m, find the force of gravitational attraction between the car and the truck.

$$\begin{aligned}F &= G \frac{m_1 m_2}{r^2} \\&= (6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2) \frac{(1.37 \times 10^3 \text{ kg})(9.92 \times 10^3 \text{ kg})}{(2.10 \text{ m})^2} \\&= 2.06 \times 10^{-4} \text{ N}\end{aligned}$$

15. A satellite is in orbit 3.11×10^6 m from the center of Earth. The mass of Earth is 5.98×10^{24} kg. Calculate the orbital period of the satellite.

$$\begin{aligned}T &= 2\pi \sqrt{\frac{r^3}{Gm_E}} \\&= 2\pi \sqrt{\frac{(3.11 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} \\&= 1.72 \times 10^3 \text{ s}\end{aligned}$$

Answer Key

Chapter 7 continued

16. The planet Venus orbits the Sun with a mean orbital radius of 1.076×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{Gm_s} \right) r^3 \\ T &= 2\pi \sqrt{\frac{r^3}{Gm_s}} \\ &= 2\pi \sqrt{\frac{(1.076 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}} \\ &= 1.92 \times 10^7 \text{ s or about 222 days} \end{aligned}$$

17. A satellite is in orbit at a distance of 6750 km from the center of Earth. The mass of Earth is 5.98×10^{24} kg. What is the orbital speed of the satellite?

$$\begin{aligned} v &= \sqrt{\frac{Gm_E}{r}} \\ &= \sqrt{\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.75 \times 10^6 \text{ m}}} \\ &= 7.69 \times 10^3 \text{ m/s} \end{aligned}$$

18. Given that the mass of Earth is 5.98×10^{24} kg, what is the orbital radius of a satellite that has an orbital period of exactly one day (assume that a day is exactly 24 hours in length)?

$$\begin{aligned} T &= 2\pi \sqrt{\frac{r^3}{Gm_E}} \\ r &= \sqrt[3]{Gm_E \left(\frac{T}{2\pi} \right)^2} \\ &= \sqrt[3]{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}) \dots} \\ &= \sqrt[3]{\dots (24 \text{ h}) \left(\frac{60 \text{ min}}{1 \text{ h}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right)} \\ &= 4.23 \times 10^7 \text{ m} \end{aligned}$$

19. The Moon has a mass of 7.349×10^{22} kg and a radius of 1737 km. How much would a 75.0-kg person weigh standing on the surface of the Moon?

$$\begin{aligned} F_{g, \text{ Moon}} &= mg_{\text{Moon}} \\ &= m \left(\frac{GM_{\text{Moon}}}{r_{\text{Moon}}^2} \right) \\ &= (75.0 \text{ kg}) \left(\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(7.349 \times 10^{22} \text{ kg})}{(1.737 \times 10^6 \text{ m})^2} \right) \\ &= 120 \text{ N} \end{aligned}$$